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HISTORY OF HOMOLOGY

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Abstract

Homology theory can be said to start with the Euler polyhedron formula, or Euler characteristic. Homology itself was developed as a away to analyse and classify manifolds according to their cycle. The term homology was first used in boilogy by the anotomist Rich Owen in 1843 when studyig the similarities of vertebrate, fins and limbs defining it as the same organ in different animals under every variety of form and function.

Keywords: Homology, Manifold, Congruent, Polyhedra.

1 INTRODUCTION

The concept of homology is traceable to Aristotle, but Belon's comparison in 1555 of a human skeleton with that of a bird expressed it overtly. Before the late 18th century, the dominant view of the pattern of organisms was the Scala naturae–even Linnaeus with his divergent hierarchical classification did not necessarily see the resulting taxonomic pattern as a natural phenomenon (Hilton.P. 1988). The divergent hierarchy,rather than the acceptance of phylogeny, was the necessary spur to discussion of homology and the concept of analogy. Lamarck, despite his proposal of evolution, attributed homology to his escalator naturae and analogy to convergent acquired characters. Significantly, it was the concept of serial homology that emerged at the end of the 18th century, although comparison between organisms became popular soon after, and was boosted by the famous Cuvier/Geoffroy Saint-Hilaire debate of the 1830s. The concepts of homology and analogy were well understood by the pre- (or anti-) evolutionary comparative anatomists before the general acceptance of phylogeny, and they were defined by Owen in 1843. The acceptance of evolution led to the idea that homology should be defined by common ancestry, and to the confusion between definition and explanation. The term 'homoplasy', introduced by Lankester in 1870, also arose from a phylogenetic explanation of homology (Hilton. P. 1988). The concept ‘‘Homology’’ was probably first used in geometry (Hoßfeld, U. and Olsson, L., 2005). Homologous angles, corners, etc. were those that had the same position in congruent or similar figures (Spemann, H., 1915). In this paper we will be using some notations like, which means “belongs to”, $\rightarrow$ which means “maps to”, $\geq$ which means “greater than or qual to”, $f(U)$ which means a function of U.

2 HISTORY OF HOMOLOGY

In biology, homology is similarity due to shared ancestry between a pair of structures or genes in different taxa. A common example of homologous structures is the forelimbs of vertebrates, where the wings of bats and birds, the arms of primates, the front flippers of whales and the forelegs of fourlegged vertebrates like dogs and crocodiles are all derived from the same ancestral tetrapod structure. Evolutionary biology explains homologous structures adapted to different purposes as the result of descent with modification from a common ancestor. The term was first applied to biology in a non-evolutionary context by the anatomist Richard Owen in 1843. Homology was later explained by Charles Darwin's theory of evolution in 1859, but had been observed before this, from Aristotle onwards, and it was explicitly analysed by Pierre Belon in 1555 (Wikipedia contributors 2021, May 13). Homology is a natural kind term. Homologues are characters of organisms that are grouped together because of a perceived unity of form, and it is assumed that this specific similarity is due to some nontrivial underlying mechanism. Homology is a concept that is supposed to play a special role for theorizing in comparative, evolutionary, and developmental biology. For this reason, accounts of homology try to get a clear picture about the nature and biological basis of homology and the role of homology for certain research areas and approaches to biology (Wagner, G. P.: 1996). In mathematics, homology is a general way of associating a sequence of algebraic objects, such as abelian groups or modules, to other mathematical objects such as topological spaces. Homology groups were originally defined in algebraic topology. Similar constructions are available in a wide variety of other contexts, such as abstract algebra, groups, Lie algebras, Galois theory, and algebraic geometry (Wikipedia contributors 2021, May 11). Homological algebra is the branch of mathematics that studies homology in a general algebraic setting. It is a relatively young discipline, whose origins can be traced to investigations in combinatorial topology (aprecursor to algebraic topology) and abstract algebra (theory of modules and syzygies) at the end of the 19th century, chiefly by Henri Poincaré and David Hilbert (Wikipedia contributors 2021, February 15). The development of homological algebra was closely intertwined with the emergence of category theory. By and large, homological algebra is the study of homological functors and the intricate algebraic structures that they entail. One quite useful and ubiquitous concept in mathematics is that of chain complexes, which can be studied through both their homology and cohomology. Homological algebra affords the means to extract information contained in these complexes and present it in the form of homological invariants of rings, modules, topological spaces, and other 'tangible' mathematical objects. A powerful tool for doing this is provided by spectral sequences (Wikipedia contributors 2021, February 15). From its very origins, homological algebra has played an enormous role in algebraic topology. Its influence has gradually expanded and presently includes commutative algebra, algebraic geometry, algebraic number theory, representation theory, mathematical physics, operator algebras, complex analysis, and the theory of partial differential equations. K-theory is an independent discipline which draws upon methods of homological algebra, as does the noncommutative geometry of Alain Connes (Wikipedia contributors 2021, February 15). According Robert Osserman,homology, in mathematics, is a basic notion of algebraic topology. Intuitively, he claims two curves in a plane or other two-dimensional surface are homologous if together they bound a region—thereby distinguishing between an inside and an outside. Similarly, two surfaces within a three-dimensional space are homologous if together they bound a three-dimensional region lying within the ambient space (Robert Osserman). There are many ways of making this intuitive notion precise. The first mathematical steps were taken in the 19th century by the German Bernhard Riemann and the Italian Enrico Betti, with the introduction of "Betti numbers" in each dimension, referring to the number of independent (suitably defined) objects in that dimension that are not boundaries. Informally, Betti numbers refer to the number of times that an object can be “cut” before splitting into separate pieces; for example, a sphere has Betti number 0 since any cut will split it in two, while a cylinder has Betti number 1 since a cut along its longitudinal axis will merely result in a rectangle. A more extensive treatment of homology was carried out in n dimensions at the beginning of the 20th century by the French mathematician Henri Poincaré, leading to the notion of a homology group in each dimension, apparently first formulated about 1925 by the German mathematician Emmy Noether (Robert Osserman). The original motivation for defining homology groups was the observation that two shapes can be distinguished by examining their holes. For instance, a circle is not a disk because the circle has a hole through it while the disk is solid, and the ordinary sphere is not a circle because the sphere encloses a two-dimensional hole while the circle encloses a one-dimensional hole. However, because a hole is ”not there”, it is not immediately obvious how to define a hole or how to distinguish different kinds of holes (Wikipedia contributors 2021, May 11). Homology was originally a rigorous mathematical method for defining and categorizing holes in a manifold. Loosely speaking, a cycle is a closed submanifold, a boundary is a cycle which is also the boundary of a submanifold, and a homology class (which represents a hole) is an equivalence class of cycles modulo boundaries. A homology class is thus represented by a cycle which is not the boundary of any submanifold: the cycle represents a hole, namely a hypothetical manifold whose boundary would be that cycle, but which is ”not there” (Wikipedia contributors 2021, May 11). Homology theory can be said to start with the Euler polyhedron formula, or Euler characteristic. This was followed by Betti’s proof in 1871 of the independence of ”homology numbers” from the choice of basis. Homology itself was developed as a way to analyse and classify manifolds according to their cycles – closed loops (or more generally submanifolds) that can be drawn on a given n dimensional manifold but not continuously deformed into each other. These cycles are also sometimes thought of as cuts which can be glued back together, or as zippers which can be fastened and unfastened. Cycles are classified by dimension. For example, a line drawn on a surface represents a 1-cycle, a closed loop or (1-manifold), while a surface cut through a three-dimensional manifold is a 2-cycle [3]. Poincare, during a period earlier, had already invented or discovered according to philosophy, the fundamental group. But he published a series of papers in which he was studying what we would call algebraic varieties, the configuration of points in higher dimensional Euclidean space given by polynomial equalities and inequalities; and he was looking again at what we might call vector fields and generalizations of vector fields on such varieties. He was led through this study to look at what we would now call the homology of these varieties. In particular, he saw the significance for the solution of such vector field problems of what were then called the Betti numbers, which determine essentially the number of holes the configuration had (Hilton.P. 1988). In particular, this word ’polyhedron’ was almost responsible for the breaking up of a beautiful friendship. What does ’polyhedron’ mean and what does it mean to classify polyhedra? To the topologist today, this is such a standard term, meaning the underlying topological space of a much more general type of combinatorial structure than that which is admitted by the geometer, the combinatorial geometer; and the classification of polyhedra, according to the topologist, is by homeomorphism generally, possibly by combinatorial equivalence, but again in a sense different from that used by the geometer. We can say that our trouble stems from Lefschetz. It’s a very key notion, this idea of homeomorphisms between polyhedra; it raised many questions, more questions than it answered, as any good mathematical notion will (Hilton.P. 1988). In connection with these Betti numbers which Poincare had established, the so called Euler characteristic was a topological invariant. The Lefschetz fixed point theorem, expressed in admittedly somewhat clumsy notation, they saw to be closely related. Indeed, the Lefschetz fixed point theorem, applied to the identity map from a space into itself, seemed to give the EulerPoincare characteristic. But, very significantly, there was also Emmy Noether in Gbttingen; she would not have been there but for Hilbert's insistence. He felt that Gottingen was a place for mathematicians and not for sexism-which, in those days, was a very special point of view. Emmy Noether recognized that what Alexandroff and Hopf were talking about and what Lefschetz had talked about should not be thought of as numbers but should be thought of as Abelian groups. So really one should credit Emmy Noether, not with the discovery of these topological invariants but with understanding their mathematical place. Thus Emmy Noether recognized the homology groups, and that the Betti numbers and torsion coefficients were merely numerical invariants of isomorphism classes of finitely-generated Abelian groups (Hilton.P. 1988).

3 CONCLUSION

The homology concept has had a long and varied history, starting out as a geometrical term in ancient Greece. Notable theorems proved using homology include the following:

- The Brouwer fixed point theorem: If f is any continuous map from the ball B^n to itself, then there is a fixed point a $a \in B^n$ with $f(a) = a$ (Wikipedia contributors 2021, May 11).

- Invariance of domain: If $U$is an open subset of R^n and $f : U \rightarrow R^n$ is an injective continuous map, then $V = f(U)$ is open and f is a homeomorphism between$U$ and V (Wikipedia contributors 2021, May 11).

- The Hairy ball theorem: any vector field on the 2-sphere (or more generally, the $2k$-sphere for any $k \geq 1$) vanishes at some point (Wikipedia contributors 2021, May 11).

- The Borsuk–Ulam theorem: any continuous function from an n-sphere into Euclidean nspace maps some pair of antipodal points to the same point. (Two points on a sphere are called antipodal if they are in exactly opposite directions from the sphere's center) (Wikipedia contributors 2021, May 11).

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