Application of concepts of topological spaces

Preston Gabriel, Mensah Kweku Emmanuel, Donkor George, Ntim Ofori Stephen, Atokple Zialesi Joy, Boamah Adomako Elizabeth, Mensah David, Okyere Aikins, Twumasi Augustine, Essuman Amankwaah Ebenezer

Department of Mathematics, KNUST, Kumasi, Ghana

ABSTRACT
In the field of mathematics topology is one of the great tools which can be applied in all aspects of science. It has three concepts: topologies on sets, manifolds, continuous functions and homeomorphism. It has various subfields namely, general topology, differential topology, algebraic topology, geometric topology etc. The concepts of topological space and its subfields can be applied in physics, biology, computer science, robotics, mechanical engineering and even cosmology.

KEYWORDS
Topology, Subsets, Shapes, Space, Deformation

INTRODUCTION
Topology is the study of shapes including their properties deformation applied to them mappings between them and configuration composed of them. Topology is often described as rubber-sheet geometry[1]. In traditional geometry, objects such as circles, triangles, planes and polyhedral are considered rigid, with well-defined distances between points and well-defined angles between edges or faces. But in topology, distances and angles are irrelevant.[1] We treat objects as if they are made of rubber, capable of being deformed. We allow objects to be bent, twisted, stretched, shrunk or otherwise deformed from one to another.
MAIN WORK

TOPOLOGICAL SPACE:
The objects that we study are topological spaces. These are sets of points on which a notion of proximity between points is established by specifying a collection of subsets called open sets. The line, the circle, the plane, the sphere, the torus, and the Mobius band are all examples of topological spaces.

TOPOLOGIES ON SETS
The term topology also refers to a specific mathematical idea central to the area of mathematics called topology. Informally, a topology tells how elements of a set relate spatially to each other. The same set can have different topologies. For instance, the real line the complex plane (which is a 1-dimensional complex vector space), and the cantor set can be thought of as the same set with different topologies.

Formally, let $X$ be a set and let $T$ be a family of subsets of $X$. Then $T$ is called a topology on $X$ if:

1. Both the empty set and $X$ are elements of $T$. 
2. Any union of elements of $T$ is an element of $T$. 
3. Any intersection of finitely many elements of $T$ is an element of $T$.

If $T$ is a topology on $X$, then the pair $(X, T)$ is called a topological space. The notation $X_T$ may be used to denote a set $X$ endowed with the particular topology $T$. Examples of topology includes the discrete topology, the trivial topology.

The members of $T$ are called open sets in $X$. A subset of $X$ is said to be closed if its complement is in $T$. A subset $X$ may be open, closed, both or neither. The empty set and $X$ itself are always both closed and open. An open set containing a point $X$ is called a neighborhood of $X$. A set with a topology is called a topological space.

CONTINUOUS FUNCTIONS AND HOMEOMORPHISMS
A function or map from one topological space to another is called continuous if the inverse image of any open set is open. If the function maps the real numbers to the real, then this definition of continuous is equivalent to the definition of continuous in calculus. If a continuous function is one-one and onto and if the inverse of the function is also continuous then the
function is called a homeomorphism and the domain of the function is said to be homeomorphic to the range. A sphere is homeomorphic to the surface of a tetrahedron[4].

MANIFOLDS
While topological spaces can be extremely varied and exotic, many areas of topology focus on the more familiar class of spaces known as manifolds. A manifold is a topological space that resembles Euclidean space near each point. More precisely each point of an n-dimensional manifold has a neighborhood that is homeomorphic to the Euclidean space of dimension n. Lines and circles, but not figure eights are one-dimensional manifolds. Two dimensional manifolds are also called surfaces.

Examples include the plane, the sphere, and the torus which can all be realized without self-intersection in three dimensions, but also the Klein bottle and real projective plane which cannot.

SUBFIELDS
There are various subfields under study in topology. These are general topology, algebraic topology, differential topology, geometric topology.

General topology also called point set topology establishes aspect of topology and investigates properties of topological spaces and concepts inherent to topological spaces. It defines the basic notions used in all other branches of topology.

Algebraic topology originated in the attempts by Poincare and Betti to construct topological invariants.[3] It tries to measure degree of connectivity using algebraic constructions such as homology and homotopy groups.

Differential topology deals with differentials manifolds

Geometric topology studies manifolds and their placement in other manifolds

APPLICATIONS
Topology can be applied in the study of DNA structure, deformation of materials (i.e. deforming a coffee cup into a doughnut shape), the study of digital images, also application of algebraic topology is to physics where it manifests in the form of higher gauge transformation Robert Ghist uses Algebraic topology to improve sense network and robotics.

BIOLOGY
Knot theory, a branch of topology is used in biology to study the effects of certain enzymes on DNA. These enzymes cut twist and reconnect the DNA causing knotting with observable effects
such as slower electrophoresis. Topology is also used in evolutionary biology to represent the relationship between phenotype and genotype. Phenotypic forms that appear quite different can be separated by only a few mutations depending on how genetic changes map to phenotypic changes during development.

**COMPUTER SCIENCE**
Topological data analysis uses techniques from algebraic topology to determine the large-scale structure of a set, for instance, determining if a cloud of points is spherical or toroidal.

**PHYSICS**
In physics, topology is used in several areas such as condensed matter physics, quantum field theory and physical cosmology. The topological dependence of mechanical properties in solids is of interest in disciplines of mechanical engineering and material science. The compressive strength of crumpled topologies is studied in attempts to understand the high strength to weight of such structures that are mostly empty space.

**ROBOTICS**
The various possible positions of a robot can be described by a manifold called configuration space. In the area of motion planning, one finds paths between two points in configuration space. These paths represent a motion of the robot’s joints and other parts into the desired location and pose.

**CONCLUSION AND RECOMMENDATION**
In conclusion topological space is a very powerful tool in mathematics which cannot be done away with, and we recommend that it should not only be studied theoretically but students should be introduced to its applications so they can apply it in real life after school.

**REFERENCES**

2. Morris A. S., (2005)*Topology Without Tears*